MATHCOUNTS® Minis September 2017 Activity Solutions

Warm-Up!

1. There are 3 cups of lemon juice and 7 cups of water or 3 + 7 = 10 cups of liquid in the container. We want to add pure water so there is 25% lemon juice in the container. We know 25% is equivalent to 1/4. We can set up the equation $3/(10 + w) = 1/4$. Cross multiplying we get $12 = 10 + w$. So $w = 12 - 10 =$ **2** cups.

2. Charlotte's seven scores have a mean of 80. So the sum of the seven scores is $80 \times 7 = 560$, and the sum of the three scores with a mean of 60 must be $60 \times 3 = 180$. Therefore, the sum of the remaining four scores is $560 - 180 = 380$, and the mean of these four scores is $380 \div 4 =$ **95**.

3. We are told the television costs $299 and the older sibling will pay $45 more than the younger sibling. This means that the other $299 - 45 = 254$ dollars will be split evenly between the two siblings. Therefore, the younger sibling will pay $254 \div 2 =$ **127** dollars.

4. The ratio of round tables to rectangular tables is 2:1, therefore 2/3 of the total number of tables are round and 1/3 are rectangular. Each round table seats 8 and each rectangular table seats 12, with the total number of seats being 336. We can set up the following equation with t representing the total number of tables: $2/3 \times 8 \times t + 1/3 \times 12 \times t = 336$. Solving for t, we get $28/3 \times t = 336$ and $t = 336 \times 3/28 =$ **36** tables.

The Problems are solved in the MATHCOUNTS® Minis video.

Follow-up Problems

5. The camel is carrying 25 small jugs. Therefore, it can carry $30 - 25 = 5$ more small jugs, or 5 small × 18 large/30 small = **3** large jugs.

6. Since Jill drives for 80%, or 4/5 the time that Jack drives the same distance, Jill's rate must be 5/4 Jack's rate. If we let r represent Jack's rate, we can write Jill's rate as $(5/4)r$. We also are told that Jill travels 10 miles per hour faster than Jack. So Jill's rate also can be written as $r + 10$. These quantities are equivalent so we can write the equation $(5/4)r = r + 10$. Solving for r, we get $(1/4)r = 10 \rightarrow r = 40$. Therefore, Jack traveled at an average rate of **40** mi/h.

7. Since Tirunesh travels at a rate of 8 meters per second, and Sally travels at a rate of 7 meters per second, it follows that Tirunesh travels 1 meter farther than Sally each second. So the first time they meet is after Tirunesh has traveled 400 meters more than Sally has traveled. Traveling 1 meter per second faster than Sally, it will take **400** seconds for Tirunesh to travel 400 meters farther than Sally.

8. This problem can be solved several ways. First let's solve it algebraically. We are told that Douglas's favorite number is a positive two-digit integer; let's call it AB where A is the tens digit and B is the units digit. That means that the value of his favorite number is 10A + B. Then a new number is created, AB7, where A now is the hundreds digit, B now is the tens digit and 7 is the units digit. The value of the new number is 100A + 10B + 7. Finally, we are told that the new number is 385 more than Douglas's favorite number. So we have $100A + 10B + 7 = 10A + B + 385$. Subtracting 10A, B and 7 from both sides yields $90A + 9B = 378$. Dividing both sides by 9 gives us $10A + B =$ **42**. This is Douglas's favorite number.