MATHCOUNTS® Minis September 2015 Activity Solutions

Warm-Up!

1. We are told that $x = y + 3$ and $y = z - 5$, which can be rewritten as $y + 5 = z$. We are asked to determine the value of $z - x$. Substituting we get $(y + 5) - (y + 3) = y + 5 - y - 3 = 5 - 3 =$ **2**.

2. If we subtract from the total the \$30 charged to hook the car to the tow truck, we see that $59.75 - 30 = \$29.75$ was the charge for the mileage. So from the school, Mr. Alman's car was towed $29.75 \div 1.75 =$ **17** miles to his house.

3. From the information given, we can write the following two equations, where x represents the weight of Tweedledee and y is the weight of Tweedledum: $x + 2y = 361$ and $2x + y = 362$. Adding the two equations, we get $3x + 3y = 723$. Dividing each side by 3, we see that the sum of their weights is $x + y =$ **241** pounds.

4. Since we don't know the dimensions of the rectangle let's call them L and W. We are told that the rectangle has an area of 108 in^2, which means that $LW = 108$. We are looking for the new area after the length and width are each increased by 1. In other words, $(L + 1)(W + 1)$. If we expand this expression we get $LW + L + W + 1$. Well, we know that $LW = 108$. We are told that the perimeter of the rectangle is 42, which means that $2(L + W) = 42 \rightarrow L + W = 21$. Substituting, we now have $LW + (L + W) + 1 = 108 + 21 + 1 =$ **130** in^2.

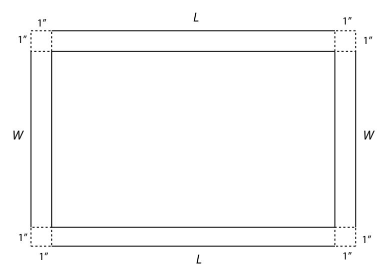
The Problem is solved in the MATHCOUNTS® Mini video.

Follow-up Problems

5. A total of $40 \times 2.15 = \$86$ would have been paid for the forty bowls of chocolate ice cream. The remaining $158.20 - 86 = \$72.20$ would have been paid for bowls of vanilla ice cream. At \$1.90 per bowl, that would mean $72.20 \div 1.90 = 38$ bowls of vanilla ice cream were sold. Thus, a total of $40 + 38 =$ **78** bowls of ice cream were sold.

6a. We are told that the perimeter of the painting is 48 inches. Since adding a frame that results in a one-inch margin around the painting essentially adds an additional 2 inches at each corner of the painting, the outer perimeter of the frame is $48 + 8 =$ **56** in.

6b. We are told that the perimeter of the painting is 48 inches. That means $2L + 2W = 48$. As the figure shows, the area of the frame is the sum of the areas of the $1\times L$ regions at the top and bottom of the painting, the $1\times W$ regions on either side and the 1×1 regions at each of the four corners. Thus, the area is of the frame is $2L + 2W + 4 = 48 + 4 =$ **52** in^2.



7. Let p represent the number of pit bulls, c is the number of chihuahuas and m is the number of mutts. The second sentence of the problem yields the following equations, where A is the total number of dogs: $p = A - 23$, $c = A - 17$, $m = A - 28$ and $A = p + c + m$. If we add the first three equations we get $p + c + m = 3A - 68$. Substituting, we get $A = 3A - 68$. We now solve to determine that the total number of dogs at the pound is $2A = 68 \rightarrow A =$ **34** dogs.

8. This problem can be solved several ways. First let's solve it algebraically. We are told that Douglas' favorite number is a positive two-digit integer; let's call it AB where A is the tens digit and B is the units digit. That means that the value of his favorite number is 10A + B. Then a new number is created, AB7, where A now is the hundreds digit, B now is the tens digit and 7 is the units digit. The value of the new number is 100A + 10B + 7. Finally, we are told that the new number is 385 more than Douglas' favorite number. So we have 100A + 10B + 7 = 10A + B +385. Subtracting 10A, B and 7 from both sides yields 90A + 9B = 378. Dividing both sides by 9 gives us 10A + B = **42**. This is Doug's favorite number.

We could also have solved the problem logically by setting up the vertical addition problem:

$$\begin{array}{r} 3\ 8\ 5 \\ +\quad A\ B \\ \hline A\ B\ 7 \end{array}$$

Notice that 5 + B = 7, so B must equal 2. We can then substitute 2 for B in the problem to get:

$$\begin{array}{r} 3\ 8\ 5 \\ +\quad A\ 2 \\ \hline A\ 2\ 7 \end{array}$$

The only integer from 1 to 9 that yields a units digit of 2 when added to 8 is 4. It follows that:

$$\begin{array}{r} 3\ 8\ 5 \\ +\quad 4\ 2 \\ \hline 4\ 2\ 7 \end{array}$$

Thus, Douglas' favorite number is **42**.