MATHCOUNTS® Problem of the Week Archive

It's Already Time to Rake! – October 21, 2024

Problems & Solutions

Suppose Leif can rake his entire yard in 2.5 hours, while his younger sister, Autumn, can do it in 4 hours by herself. How many minutes will it take the two of them to rake the entire yard if they are working together, to the nearest whole number?

> *If they are working together, then they will both be working for the same amount of time, x. Since we want our answer in minutes, let's change the amounts of time it will take them to do the entire job on their own to 150 minutes and 240 minutes. This means that Leif will get x/150 of the job done in x minutes, while Autumn will get x/240 of the job done in x minutes. However, together their partial jobs should add up to the entire job. So, x/150 + x/240 = 1. Getting a common denominator of 1200, we see this is the same as (8x)/1200 + (5x)/1200 = 1, which is the same as 13x/1200 = 1. Multiplying both sides by 1200/13, we see that x ≈ **92** minutes (rounded to the nearest whole number), which is the length of time it will take for the two of them to rake the yard if working together.*

Once Leif and Autumn have all of the leaves in their yard in a huge pile, they decide to do a little math with their pile of leaves. They notice that all of the leaves are yellow and/or red. So, they decide that Leif will count the leaves with any red (5550 leaves) and Autumn will count the leaves with any yellow (6820 leaves). Then, they get their other little brother, Oakie, to come out and count the leaves that have both yellow and red on them (4370 leaves). Using their reported numbers, how many leaves are in the pile?

> *If there are 6820 leaves with any yellow and 5550 leaves with any red, we can't just add these numbers together. We must remember that 4370 of these leaves were counted twice because they had both yellow and red on them. Therefore, there were 6820 + 5550 − 4370 = **8000** leaves.*

Leif bets Autumn that she can't reach into the middle of the pile and pull out three leaves with yellow on them in her first three picks, without replacement. Assuming the different kinds of leaves were all mixed up together again and randomly distributed throughout the pile, what is the probability that she will be successful? Express your answer as a decimal to the nearest hundredth.

> *We know there are 8000 leaves in the pile and 6820 of them will give Autumn the outcome she desires – a leaf with some yellow on it. Therefore, the probability of pulling out three "good" leaves in a row, without replacement, is (6820/8000) × (6819/7999) × (6818/7998) = **.62**, to the nearest hundredth.*

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