MATHCOUNTS® Problem of the Week Archive

Barbecues – June 2, 2025

Problems & Solutions

Seth is having a barbecue to celebrate the end of the school year. There will be a total of 18 people at the barbecue. He has set up 3 tables that each seat 6 people. How many distinct sets of three groups of six people are possible?

Looking at the three tables, the first table has 18(17)(16)(15)(14)(13)/(6!) = 18,564 possible combinations of people. The second table has (12)(11)(10)(9)(8)(7)/(6!) = 924 possible combinations. This leaves the third table, which has (6!)/(6!) = 1 possible combination. Each of the possible combinations above has an equal likelihood of being paired with each of the other tables' combinations, so we must multiply 18,564 by 924 by 1. However, since we are just looking at how many sets of three tables can be created, order does not matter (i.e. set 1, 2, 3 is the same as set 2, 1, 3). So, we must also divide by 6, which gives us (18,564)(924)(1)/(6) = ***2,858,856*** *possible sets of three groups with 6 people in each group.*

Seth made 3 gallons of lemonade for the barbecue. When he realizes that the lemonade is exactly half gone, he gets worried that he will run out before the end of the party. To prevent this, he decides that he should add the 0.10 gallons of lemon juice that he has left to the remaining lemonade and then fill the rest of his 3-gallon container with water. His original recipe was 20% lemon juice. Once he has topped-off his container as planned, what percent of the new lemonade mixture is lemon juice? Express your answer to the nearest tenth.

When Seth's lemonade is 1/2 gone, there is 1.5 gallons remaining. Since the original mixture is 20% lemon juice, there are (1.5)(0.2) = 0.3 gallons of lemon juice remaining. To that, he is going to add the 0.10 gallons of lemon juice he has, meaning that the new mixture will contain 0.4 gallons of lemon juice. The new mixture will be (0.4/3)(100) = ***13.3****% lemon juice, to the nearest tenth.*

After eating, Seth and one of his guests play a lawn game. In the game, they take turns trying to toss a beanbag into baskets worth 1 or 2 points (if the beanbag doesn't make it into either basket, no points are earned on that turn.) During each turn, a player gets one toss. The first person to earn 10 points wins. Being the polite host he is, Seth let his friend toss first. By the end of the game, each player had tossed the beanbag 6 times. If Seth won and the final score was 7-to-10, how many scoring sequences would produce this result?

The possible scoring combinations for Seth's 10 points are 2, 2, 2, 2, 2, 0, which has 6 possible sequences, and 2, 2, 2, 2, 1, 1, which has 15 possible scoring sequences. Thus, Seth's 10 points have a total of 15 + 6 = 21 possible scoring sequences. Seth's friend's 7 points have three possible scoring combinations: 2, 2, 2, 1, 0, 0, which has 60 possible sequences; 2, 2, 1, 1, 1, 0, which has 60 possible sequences; and 2, 1, 1, 1, 1, 1, which has 6 possible sequences. Thus, Seth's friend has 60 + 60 + 6 = 126 possible scoring sequences. Together, there are a total of 21(126) = ***2646*** *possible scoring sequences.*

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