MATHCOUNTS® Problem of the Week Archive

New Year's Resolution – January 6, 2025

Problems & Solutions

Jean made a New Year's resolution to get in shape. She decides to run for 30 minutes on Tuesdays, Thursdays, Saturdays and Sundays. If Jean plans to run at an average speed of 6 mi/h, how many miles will she run during the month of February 2025?

There are 4 Tuesdays, 4 Thursdays, 4 Saturdays and 4 Sundays in February, and since 30 minutes = 0.5 hours, she will run for 0.5 × 4 × 4 = 8 hours during February. Using the formula distance = rate × time, we find that Jean will run 8 × 6 = ***48*** *miles in February.*

After some consideration, Jean decides that it might be better to run at an average speed of 5.0 mi/h on her first day of running in February and then increase her average speed by 0.1 mi/h each day she runs. How many fewer miles will she run than if she were to run at an average speed of 6.0 mi/h each time she runs?

Let's start with some calculations using distance = rate × time:

5.0 × 0.5 = 2.5 miles

5.1 × 0.5 = 2.55 miles

5.2 × 0.5 = 2.6 miles

5.3 × 0.5 = 2.65 miles

We can see that these calculations form an arithmetic sequence, in which each time Jean increases her speed by 0.1 mi/h, her distance increases by 0.05 miles. To find the sum of the number of terms in an arithmetic sequence, you find the sum of the first term and the last term, multiply this sum by the number of terms, and then divide by 2. So, let's find the last term (the number of miles Jean would run on her last running day in February).

$a_1 + d(n - 1) = a_n$ *(where d = common difference,* a_1 *= first term in the sequence,* a_n *= term we're looking for)*

$2.5 + 0.05(16 - 1) = 3.25$

Thus, the total number of miles Jean would run is $[(2.5 + 3.25)(16)] \div 2 = 46$ *miles.*

This is 48 – 46 = ***2*** *miles fewer than if she ran at an average rate of 6 mi/h each time she ran in February.*

By following the new plan, by what percent will Jean have increased her average speed from the first running day in February to the last running day in February?

If Jean starts February at an average speed of 5.0 mi/h and increases her speed by 0.1 mi/h each time she runs, the last day she runs in February, she will run at an average speed of 5.0 + 0.1(16 – 1) = 6.5 mi/h. That is an increase of 6.5 – 5.0 = 1.5 mi/h, which is a percent increase of (1.5/5.0) × 100 = ***30****%.*

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