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Activity Sheet for the October, 2013, MATHCOUNTS Mini

Try these problems before watching the lesson.

1. Expand the product $(x + 2)(x - 7)$.
2. Expand the product $(3 - y)(8 - y)$.
3. Find all values of r such that $r^2 + 3r - 70 = 0$.
4. What is the sum of the values of x for which $x^2 - 13x + 40 = 0$?

First Problem: The quadratic equation $x^2 + bx + c = 0$ has real roots 4 and –6. What is the value of $b + c$?

Second Problem: In the equation $(x - 8)(x - k) = x^2 - 5kx + m$, k and m are positive integers. If the equation is true for all values of x, what is the value of m?

Third Problem: If $x^2 - 45b^2 = 4xb$, what is the largest possible ratio of b to x?

Fourth Problem: What is the sum of all real numbers x such that

$4^x - 6 \cdot 2^x + 8 = 0$?

Follow-up Problems

5. The quadratic equation $2x^2 + bx + c = 0$ has real roots 4 and –6. What is the value of $b + c$? (Note: This problem is *not* exactly the same as the first problem in the video!)

6. The quadratic equation $x^2 + bx + 24 = 0$ has 2 as one of its roots. What is the other root?

7. Find all values of x such that $\sqrt[3]{x^2} - 3\sqrt[3]{x} = 28$.

8. Find all solutions to the equation $t^4 + 18 = 11t^2$.

9. Find all values of x such that $4^x = 33 \cdot 2^{x-1} - 8$.

Further Exploration

10. In the video, Richard suggested that we might be able to find the sum and the product of the roots of a quadratic just by looking at the quadratic's coefficients, *without even finding the roots*. Explore this idea by finding the roots of each of the following quadratics, and then comparing each quadratic's coefficients to the sum and product of its roots. Notice anything interesting? Can you explain the patterns you find?

    (a) $x^2 + 2x - 35$.

    (b) $2x^2 - 5x - 3$.

    (c) $2x^2 + 4x - 70$.

    (d) $12x^2 - 11x + 2$.

Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).