Activity Sheet for the December, 2009, MATHCOUNTS Mini

1. Find 5 ordered pairs (x, y) that satisfy the equation $x + \frac{1}{y} = 1$.

2. Find all ordered pairs of integers (x, y) that satisfy the equation $x + \frac{1}{y} = 1$.

If $x + \frac{1}{y} = 1$ and $y + \frac{1}{z} = 1$, then what is the value of the product xyz?

3. If $x + \frac{1}{y} = 2$ and $y + \frac{1}{z} = \frac{1}{2}$, then what is the value of the product xyz?

4. If $x + \frac{1}{y} = 3$ and $y + \frac{1}{z} = \frac{1}{3}$, then what is the value of the product xyz?

5. If $x + \frac{1}{y} = 4$ and $y + \frac{1}{z} = \frac{1}{4}$, then what is the value of the product xyz?

6. See anything interesting in the answers to the previous three questions? Will the pattern continue?

Further Exploration

7. If $x + \frac{1}{y} = 2$ and $y + \frac{1}{z} = 1$, then is there only one possible value of xyz?

8. If $x + \frac{1}{y} = 1$ and $y + \frac{1}{z} = 1$, then must we also have $z + \frac{1}{x} = 1$?

Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).