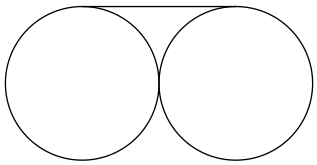
Activity Sheet for the October, 2010, MATHCOUNTS Mini

Warm-Up!

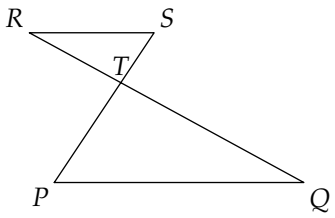
Try these problems before watching the lesson.

1. What is the circumference of a circle with radius 12 inches?

2. The circles in the diagram below are tangent, meaning they only touch at one point. Each circle has radius 0.4 inches. The line segment is tangent to both circles, and each endpoint of the segment is on one of the circles. What is the length of the line segment in inches?

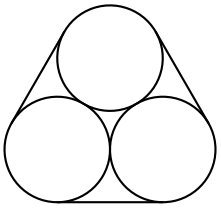


3. In the diagram below, triangles RTS and QTP are similar. If $PQ = 9$, $RS = 6$, and $SP = 10$, then what are PT and ST?

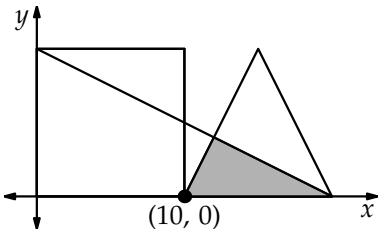


The Problem

First Problem: A belt is drawn tightly around three circles of radius 10 cm each, as shown below. What is the length of the belt in cm?

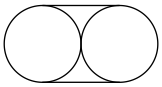


Second Problem: A square and isosceles triangle of equal height are side-by-side, as shown, with both bases on the x-axis. The lower right vertex of the square and the lower left vertex of the triangle are at $(10, 0)$. The side of the square and the base of the triangle on the x-axis each equal 10 units. A segment is drawn from the top left vertex of the square to the farthest vertex of the triangle, as shown. What is the area of the shaded region?

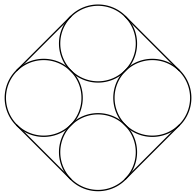


Follow-up Problems

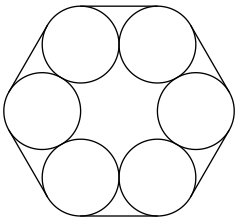
4. A belt is drawn tightly around two circles of radius 10 cm each, as shown below. What is the length of the belt in cm?



5. A belt is drawn tightly around four circles of radius 10 cm each, as shown below. What is the length of the belt in cm? (The centers of the circles are the vertices of a square.)

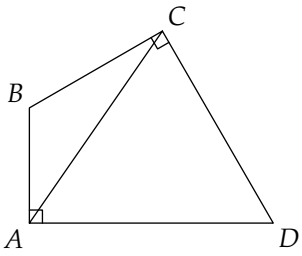


6. A belt is drawn tightly around six circles of radius 10 cm each, as shown below. What is the length of the belt in cm? (The centers of the circles are the vertices of a regular hexagon.)



7. $ABCD$ is a trapezoid with $\overline{AB} \parallel \overline{CD}$. The diagonals of the trapezoid intersect at point E. If $AB = 4$, $CD = 8$, and the height of the trapezoid is 9, then what is the area in square units of $\triangle ABE$?

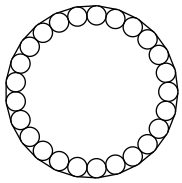
8. In triangle ABC shown below, $\angle ABC = 120°$, $AB = 3$, and $BC = 4$. If lines perpendicular to $\overline{AB}$ at A and to $\overline{BC}$ at C meet at D, then find CD. (*Source: AMC 12*) Here's a Hint from Harvey: *I see a 30-60-90 triangle!*



9. What is the last thing that Harvey says in the lesson?

Further Exploration

10. Write a computer program that generates diagrams like those in the band-around-the-circles problems for any number of circles. For example, if you enter 25 into your program, your program should output 25 circles whose centers form a regular polygon, such that each circle is tangent to its neighbors. The program should also attach the band to the outside circles. The output should look like below. (Note that the outer band is not a circle! It has straight portions between the circles, and curved portions on the circle.)



Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).