Activity Sheet for the March, 2017, MATHCOUNTS Mini

Warm-Up!

Try these problems before watching the lesson.

1. If $f(n) = n^2 + n + 17$, what is the value of $f(11)$?

2. If $S(n)$ is a function that returns the sum of the first n positive integers, then what is $S(20) - S(19)$?

3. The formula $d = 16t^2$ is used to calculate the distance, d, in feet, a free falling object, starting from rest, will travel in t seconds. How many seconds will it take for a ball, starting from rest, to free fall from a height of 64 feet to the ground?

4. Define $E(a, b, c) = a \times b^c$. What positive value of r is the solution to the equation $E(r, r, 3) = 625$?
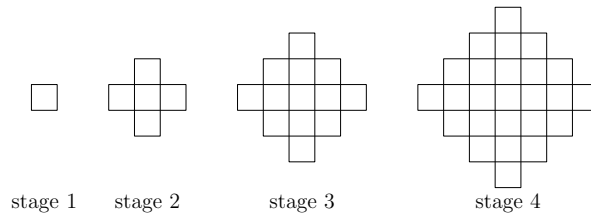
The Problems

First Problem: A function $f(x)$ is defined for all positive integers. If $f(a) + f(b) = f(ab)$ for any two positive integers a and b and $f(3) = 5$, what is $f(27)$?

Second Problem: Starting at the origin, a bug crawls 1 unit up, 2 units right, 3 units down and 4 units left. From this new point, the bug repeats this entire sequence of four moves 2015 more times, for a total of 2016 times. The coordinates of the bug's final location are (a, b). What is the value of $a + b$?

Third Problem: The function $f(n) = a \cdot n! + b$, where a and b are positive integers, is defined for all positive integers. If the range of f contains two numbers that differ by 20, what is the least possible value of $f(1)$?

Follow-up Problems

5. The point $(0, 8)$ lies on the graph of the curve defined by $y = c \cdot 2^x$. What is the value of y when $x = 2$?

6. If $f(x) = x^2 + 5$ and $g(x) = 2(f(x))$, what is the greatest possible value of $f(x + 1)$ when $g(x) = 108$?

7. Suppose $f(x) = x^2 + 12$. If $m > 0$ and $f(3m) = 3(f(m))$, what is the value of m?

8. At each stage, a new square is drawn on each side of the perimeter of the figure in the previous stage. How many unit squares will be in stage 10?

stage 1   stage 2   stage 3   stage 4

Wow! Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).