MATHCOUNTS®

Faster Arithmetic Methods

Warm-Up!

Try these problems before watching the lesson.

1. What is $40\% \times \frac{2}{3} \times 24 \div 0.8$?

2. What is the value of 1 × 12 + 2 × 11 + 3 × 10 + 4 × 9 + 5 × 8 + 6 × 7 + 7 × 6 + 8 × 5 + 9 × 4 + 10 × 3 + 11 × 2 + 12 × 1?

3. Mac has 25 marbles, of which 20% are red. Thayer has 20 marbles, of which 75% are not red. What is the absolute difference between the numbers of red marbles they have?

4. What is the value of the sum $\frac{1}{9}+\frac{2}{9}+\frac{3}{9}+\frac{4}{9}+\frac{5}{9}+\frac{6}{9}+\frac{7}{9}+\frac{8}{9}$?

5. What is the value of the sum 0.49 + 0.53 + 0.55 + 0.47 + 0.48? Express your answer as a decimal to the nearest hundredth.

The Problems

Take a look at the following problems and follow along as they are explained in the video.

6. What is the value of the sum $\frac{1}{87}+\frac{2}{87}+\frac{3}{87}+\frac{4}{87}+\cdots+\frac{84}{87}+\frac{85}{87}+\frac{86}{87}$?

7. What is the value of 1 × 9 + 2 × 99 + 3 × 999 + 4 × 9999 + 5 × 99999?

8. What is the value of 55 × 33 – 15?

Piece It Together

Use the skills you practiced in the warm-up and strategies from the video to solve the following problems.

9. What is $12 \times 37 + 12 \times 7 + 12 \times 6$?

10. What is the value of $2 \times 6^3 + 6^2 - 7 \times 6^2$?

11. Audra adds the numbers 2018 and 22, then multiplies the result by 2 and assigns this value to a. Beto multiplies the numbers 2018 and 2, then adds 22 to the result and assigns this value to b. What is the value of $a - b$?

12. What is the value of $(1 + 3 + 5 + \ldots + 2017) - (2 + 4 + 6 + \ldots + 2016)$?

13. What is the greatest prime factor of $3^7 - 27$?

Optional Extension

To extend your understanding and have a little fun with math, try the following activities.

Option 1

When asked to multiply two multi-digit numbers without a calculator, many people instinctively compute the value using the standard algorithm—multiplying each digit from one number by each digit from the other number, systematically, and adding up the products. For example:

$$\begin{array}{r} 34 \\ \times 12 \\ \hline 68 \\ +34 \\ \hline 3780 \end{array}$$

Can you explain or prove, using properties of multiplication, why this algorithm works? Can you write a general statement or proof for multiplying two arbitrary two-digit numbers AB and CD where A, B, C and D represent digits and A and C are not zero?

Option 2

Using factoring, find the value of the following expression:

$$\frac{2017^2 + 11(2017) - 42}{2014}$$