*This practice plan was created by **Taren Long**, a math teacher and coach at Chesapeake Public Charter School. Taren created numerous free resources for MATHCOUNTS coaches in her role as the 2020-2021 DoD STEM Ambassador for MATHCOUNTS. Find more resources and information at **dodstem.us**.*

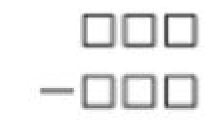
Addition and Subtraction Strategies

Warm-Up!

Coach instructions: *Give students around 10 minutes (2 minutes per problem) to go through the warm-up problems.*

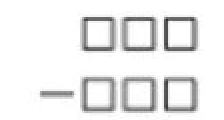
Try these problems before watching the lesson.

1. Given that the digits 1, 2, 3, 4, 5 and 6 are placed in the boxes shown, what is the greatest possible positive difference that can be obtained?



We need to start with creating the largest difference in the hundreds column, so we can start by placing a 6 and 1 in each of those boxes. Then we look at the tens column and place the remaining largest and smallest numbers in each of those respectively. Finally, we place the remaining two in the ones column. This gives us 654 as the first number and 123 as the second number. The difference between those is 531.

2. Given that the digits 1, 2, 3, 4, 5 and 6 are placed in the boxes shown, what is the smallest possible positive difference that can be obtained?



To create the smallest possible positive difference, the tens and ones digits of the second number should be as large as possible (65), and the tens and ones digits of the first number should be as small as possibe (12). That leaves 4 and 3 remaining, so to make a positive difference, the first number needs to begin with a 4. The two numbers are 412 and 365, and their difference is 47.

3. If n and m are whole numbers, $99 < n < 401$, and $19 < m < 81$, what is the greatest possible value of $\frac{n}{m}$?

For the greatest possible value of $\frac{n}{m}$, we need the greatest value of *n* and the least value of *m*. If $n < 401$ and *n* is a whole number, then the greatest value of *n* is 400. If $19 < m$ and *m* is a

whole number, then the least value of *m* is 20. The greatest possible value of $\frac{n}{m}$ is $\frac{400}{20} = 20$.

4. What is the sum of the three missing digits in the subtraction problem $5\_,661 - \_2,83\_ = 17,825$?

Starting with the ones column, we are taking something away from 1 and getting 5, so we will have to carry the 1 from the tens column to the ones column to get 11 − _ = 5, so the ones blank must be 6. Our remaining information is $5\_,661 - \_2,836 = 17,825$. In the hundreds column, a difference of 8 requires you to have carried from the thousands column. In the thousands column, _ − 2 = 7 suggests the digit should be 0 to then become a 9 when the digit is carried. Thus, in the difference $50,661 - \_2,836 = 17,825$ the remaining digit must be 3. Adding the digits 3, 0 and 6 gives a sum of 9.

5. The integer 12,345 can be expressed as the sum of two prime numbers in exactly one way. What is the larger of the two primes in this sum?

Note that with the distinct exception of 2, all primes are odd. 12,345 is odd, yet an odd number plus another odd number will always yield an even number as the result. Thus, one of the primes must be even, and is equal to 2. The remaining (larger) prime is $12,345 - 2 = 12,343$.

The Problems

<u>Coach instructions:</u> *After students try the warm-up problems, play the video and have them follow along with the solutions.*

Take a look at the following problems and follow along as they are explained in the video.

6. For positive integers *x* and *y*, $\frac{1}{x} + \frac{1}{y} = \frac{5}{12}$, What is the least possible value of $x + y$?

Solution in video. Answer: 10.

7. Using each of the digits 3, 5, 7, 9 exactly once in the expression of the sum below, what is the larger of the two common fractions that would give a sum between 0.75 and 1?

$$\frac{\square}{\square} + \frac{\square}{\square}$$

Solution in video. Answer: 5/9.

8. What is the difference between the sum of the first 400 even counting numbers and the sum of the first 400 odd counting numbers?

Solution in video. Answer: 400.

Coach instructions: *After watching the video, give students 10 minutes to try the next four problems.*

Piece It Together

Use the skills you practiced in the warm-up and strategies from the video to solve the following problems.

9. For positive integers *x* and *y*, $\frac{1}{x}-\frac{1}{y}=\frac{1}{12}$. What is the least possible value of $x+y$?

Values of *x* and *y* that could have 12 as a common denominator are 2, 3, 4 and 6. Trying out different combinations, we find two combinations that work: $\frac{1}{3}-\frac{1}{4}=\frac{4}{12}-\frac{3}{12}=\frac{1}{12}$ and $\frac{1}{4}-\frac{1}{6}=\frac{3}{12}-\frac{2}{12}=\frac{1}{12}$. 3 and 4 have the least possible sum, which is 7.

10. Use the digits 1-6, at most one time each, to create an equation where *x* has the greatest possible value.

$$\square\square + x = \square\square$$

To ensure *x* has the greatest possible value, the sum would need to contain the largest digits possible, 65. For *x* to be largest, the number being added to it should be as small as possible, so it must be 12. 12 + *x* = 65, and *x* must be 53.

11. The denominator of a fraction is 2 more than its numerator. The reciprocal of this fraction is equal to the fraction itself. What is the sum of the numerator and denominator?

For the fraction to equal its reciprocal, the numerator *x* cannot be positive. Testing some values of *x* yields $\frac{1}{3}\neq\frac{3}{1};\frac{2}{4}\neq\frac{4}{2};\frac{3}{5}\neq\frac{5}{3}$, etc. *x* cannot be 0 because then the reciprocal would be 2/0, which is impossible. Testing *x* = −1, we get $\frac{-1}{-1+2}=\frac{-1+2}{-1}$, which simplifies to $\frac{-1}{1}=\frac{1}{-1}$. Since −1 = −1, this satisfies the conditions. Thus, the sum of the numerator and denominator is −1 + 1 = 0.

12. The fraction $\frac{x}{5}$ does not change its value when 3 is added to both its numerator and its denominator. What is the value of *x*?

We can represent the equality between the expressions with the equation: $\frac{x}{5}=\frac{x+3}{8}$. We can solve with cross multiplication and solving the equation, or we can rewrite each fraction such that they have a common denominator of 40, $\frac{8x}{40}=\frac{5x+15}{40}$. Either way, there is equality between 8*x* and 5*x* + 15. Solving for *x*, we get *x* = 5.

Optional Extension

Coach instructions: *Once your students have completed the problems and feel they have a comfortable understanding of the concept, have them consider the following task.*

To extend your understanding and have a little fun with math, try the following activities.

Using the digits 1-9, at most one time each, fill in the boxes to make the smallest (or largest) difference. Try this first using only positive numbers and positive differences. How would your solution change if you allow negative integers? What types of differences can you make? Are there any differences you cannot obtain?

$$\pm\square\square - \pm\square\square$$

Considering only positive integers and differences, the largest difference that can be obtained is 98 – 12 = 86. The smallest difference is obtained in a variety of ways, such as 81 – 79, or 71 – 69, both of which equal 2. If you consider negative integers, the largest difference becomes 97 – (–86) = 183.