Activity Sheet for the December, 2017, MATHCOUNTS Mini

Warm-Up!

Try these problems before watching the lesson.

1. Samantha walks 5 miles north, 2 miles east, and then one mile north to point P. How many miles from her starting point is point P? Express your answer in simplest radical form.

2. When leaned against a vertical structure, a straight ladder can be used safely if its top is no more than 4 feet above the base of the structure for every foot that the bottom of the ladder is away from the base. How high can a 22-foot ladder safely reach up a vertical structure? Express your answer as a decimal to the nearest tenth.

3. Two 8.5-inch by 11-inch sheets of paper are lying flat on an otherwise unoccupied 2-foot by 3-foot tabletop. Exactly 700 in^2 of the table are not covered by the sheets of paper. What is the area of the overlap of the two sheets, in square inches?

4. Bill, Melissa and three friends are seated at a round table. Melissa does not sit next to Bill. Assume that two arrangements are considered the same if they are rotations of each other. How many distinct seating arrangements are possible?

The Problems

First Problem: Nina's two dogs, Biter and Nipper, normally eat an entire bag of dog food kibbles in 10 days. She has fed them both for 7 days when Biter breaks a tooth and stops eating the hard food. It takes Nipper 9 more days to finish the bag. What is the ratio of the number of days it would take Biter to eat the whole bag alone to the number of days it would take Nipper to eat the whole bag alone? Express your answer as a common fraction.

Second Problem: Eddie and Missy are swimming laps in parallel lanes of a swimming pool at different constant speeds. They start simultaneously at opposite ends of the pool. They first pass each other when Eddie has swum 72 feet. Both turn back when they reach the opposite ends, and they next pass each other when Eddie is 40 feet from Missy's starting point. What is the length of a lap?

Follow-up Problems

5. Two sides of a right triangle are 9 units and 12 units. What is the numerical value of the product of the two possible lengths of the third side? Express your answer in simplest radical form.

6. Three-fifths of the way up a hill, Jack and Jill realized that they had forgotten their bucket. Jill continued up the hill, while Jack went back down the hill to get the bucket. Two minutes after turning back, Jack reached the bottom of the hill at the exact same time that Jill reached the top. If the total distance from the bottom to the top of the hill is 1260 feet, what is the absolute difference in Jack's downhill speed and Jill's uphill speed, in feet per second? Express your answer as a decimal to the nearest tenth.

7. Two intersecting circles have a common chord of length 16 ft, and their centers lie on opposite sides of the chord. The radii of the circles are 10 ft and 17 ft respectively. Express the distance between the centers of the circles in feet.

8. A 36-inch rope is cut into three pieces. One piece is five inches longer than another, and one piece is twice as long as another. What is the sum of the possible lengths of the longest piece? Express your answer as a decimal to the nearest tenth.

Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).