MATHCOUNTS®

Counting Shapes in a Composite Figure

Warm-Up!

Coach instructions: *Give students around 9 minutes (3 minutes per problem) to go through the warm-up problems.*

Try these problems before watching the lesson.

1. How many different squares of any size are there on the 3 × 3 shown in Figure 1

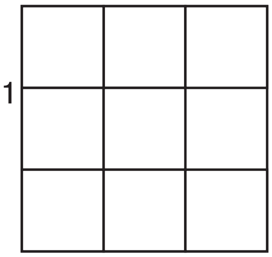
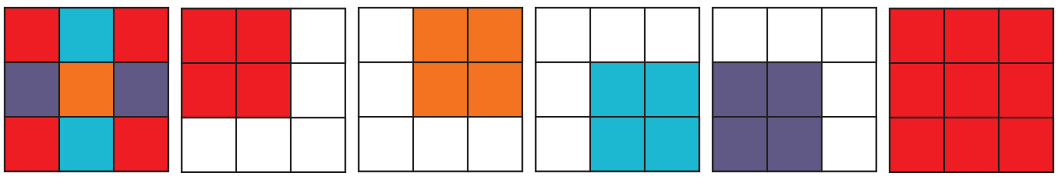


Figure 1

There are three different sized squares in the figure, 1 × 1, 2 × 2 and 3 × 3. There are 9 squares of size 1 × 1, 4 of size 2 × 2 and 1 of size 3 × 3, for a total of 9 + 4 + 1 = 14 squares.



2. How many rectangles of any size are in the grid shown in Figure 2?

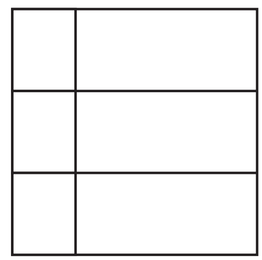
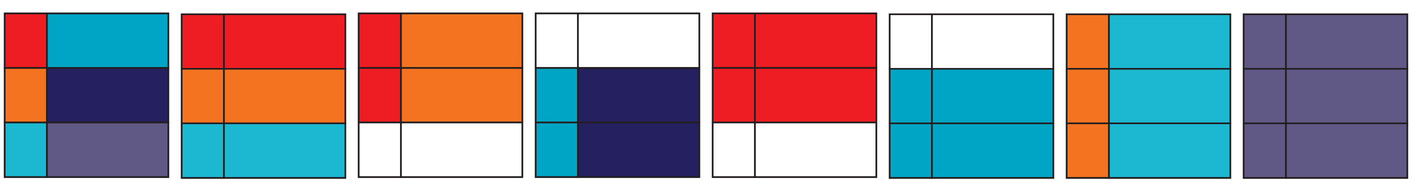


Figure 2

There are 6 rectangles that are one row high by one column wide. There are 3 rectangles that are one row high by two columns wide. There are 4 rectangles that are two rows high by one column wide. There are 2 rectangles that are two rows high by two columns wide. There are 2 rectangles that are three rows high by one column wide. Finally, there is 1 rectangle that is three rows high by two columns wide. The total number of rectangles is 6 + 3 + 4 + 2 + 2 + 1 = 18 rectangles.



3. How many triangles of any size are in the figure shown in Figure 3?

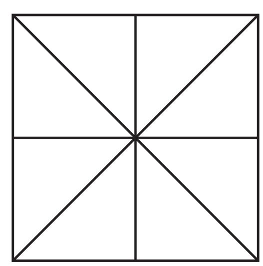
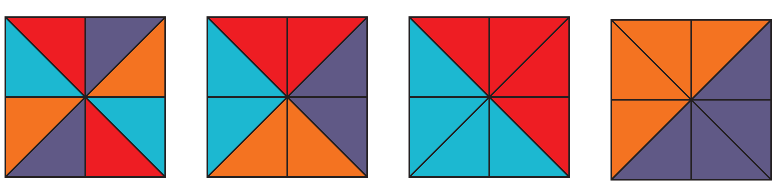


Figure 3

If we break this composite figure up into its smallest triangle parts, we see there are 8 of these small triangles. We can then look at triangles that are composed of two small triangles and count 4 of these. There are no such triangles that are composed of three small triangle. So, the next sized triangle is composed of four small triangles and we count 4 of these. The total number of triangles is therefore 8 + 4 + 4 = 16 triangles.

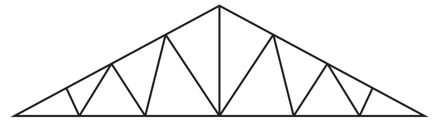


The Problems

Coach instructions: After students try the warm-up problems, play the video and have them follow along with the solutions.

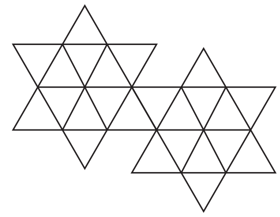
Take a look at the following problems and follow along as they are explained in the video.

4. How many triangles of any size are in the Belgian truss shown?



Solution in video. Answer: 23 triangles.

5. How many triangles of any size are in the fiugre shown here?



Solution in video. Answer: 40 triangles.

Piece It Together

Coach instructions: After watching the video, have student go back and finish any warm-up problems they didn't have time to complete or solved incorrectly. Then give students another 9 minutes to try the next three problems.

Use the skills you practiced in the warm-up and strategies from the video to solve the following problems.

6. How many squares of any size in this figure consisting of adjacent unit squares surrounding a larger square shown in Figure 4?

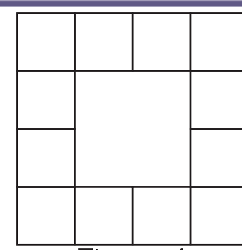
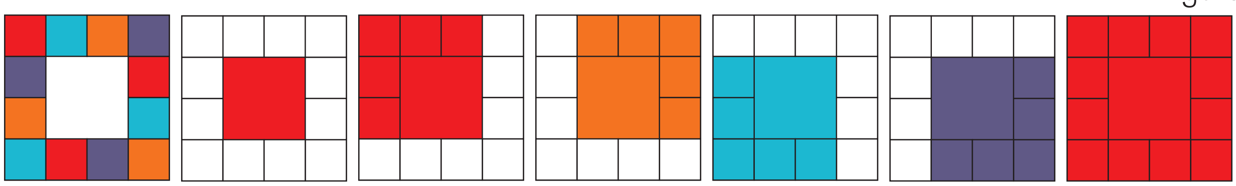


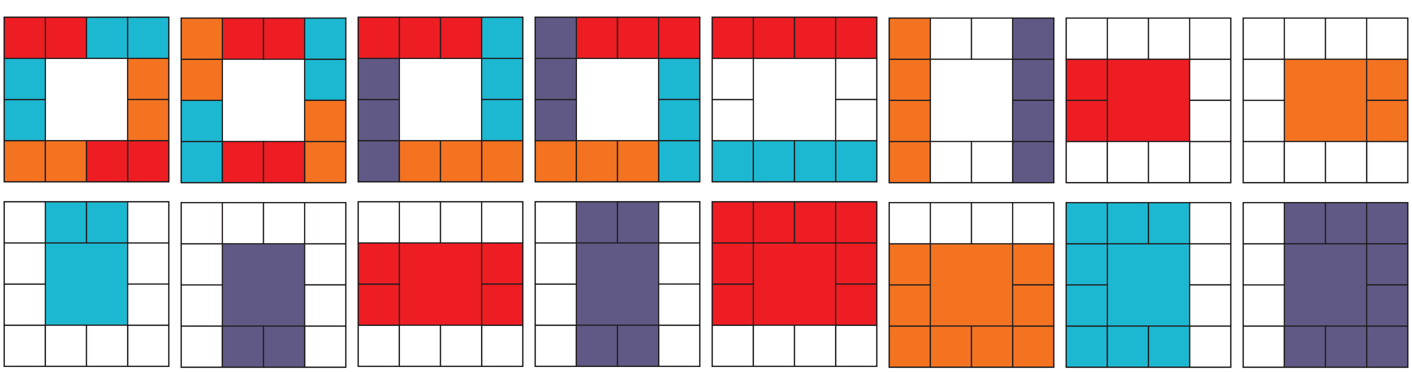
Figure 4

There are 12 1×1 squares, there is 1 2×2 square, there are 4 3×3 squares and there is 1 4×4 square. In total, we count 12 + 1 + 4 + 1 = 18 squares.



7. Using Figure 4 again, how many rectangle of any size are there?

This solution builds on the previous problem. Since squares are also rectangles, all 18 will be counted again. We now need to count the rectangles that are not squares. There are 12 1×2 or 2×1 rectangles, 8 1×3 or 3×1 rectangles, 4 1×4 or 4×1 rectangles, 4 2×3 or 3×2 rectangles, 2 2×4 or 4×2 rectangles and 4 3×4 or 4×3 rectangles. Thats a total of 12 + 8 + 4 + 4 + 2 + 4 = 34 rectangles that we didn't count in the last problem. In total we have 34 + 18 = 52 rectangles.



8. How many triangles of any size are in the figure shown here?

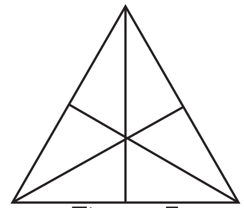
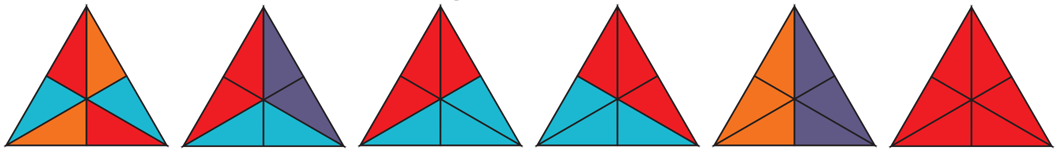


Figure 5

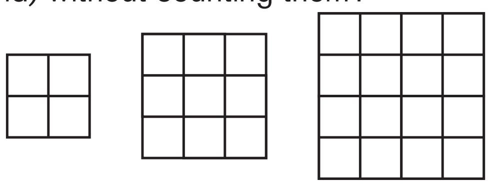
There are 6 small, non-composite triangles. There are 3 triangles that are composed of two triangles. There are 6 triangles that are composed of three triangles. There are no triangles that can be made out of four or five triangles. Finally, there is 1 triangle that is composed of all six small triangles. In total, 6 + 3 + 6 + 1 = 16 triangles.



Optional Extension

To extend your understanding and have a little fun with math, try the following activities.

Return to your thinking about the 3 × 3 grid from Problem 1. Consider a 2 × 2 grid and also a 4 × 4 grid as shown below. How does the total number of squares of any size change as the dimensions of the grid is increased? Can you come up with a rule that would help you calculate the number of squares of any size in a 7 × 7 grid (or any $n \times n$ grid) without counting them?



In order to create a rule, we need to establish a pattern as we count squares in larger and larger grids. Setting up a table as shown below helps us establish this pattern. We notiec, the number of squares in a 2 × 2 grid is $2^2 + 1^2 = 4 + 1 = 5$, the number of squares in a 3 × 3 grid is $3^2 + 2^2 + 1^2 = 9 + 4 + 1 = 14$ and the number of squares in a 4 × 4 grid is $4^2 + 3^2 + 2^2 + 1^2 = 16 + 9 + 4 + 1 = 30$. The pattern is a series of square numbers. We can use this to determine that the number of squares in a 7 × 7 grid would be $7^2 + 6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2 = 49 + 36 + 25 + 16 + 9 + 4 + 1 = 140$. In general, for an $n \times n$ grid, the number of squares will be $n^2 + (n - 1)^2 + (n - 2)^2 + \cdots + 1^2$. Establishing a pattern is a good strategy to practice in MATHCOUNTS problems that seem to ask for intimidatingly long answers or calculations.

	# in 2 × 2 grid	# in 3 × 3 grid	# in 4 × 4 grid
1 × 1 squares	4	9	16
2 × 2 squares	1	4	9
3 ×3 squares	0	1	4
4 × 4 squares	0	0	1

Where are these questions from?

1. *From 1995 School Handbook, #WU5-7*
2. *From 2016 State Sprint, #2*
3. *From 2016 School Handbook, #222**
4. *From 2017 Chapter Sprint, #12*
5. *From 2017 State CDR, #61*
6. *From 2017 Chapter CDR, #10*
7. *From the 2018 School Handbook, #226*
8. *From 2017 School Sprint, #21*
**problem modified*