*This practice plan was created by **Tyler Erb**, a math teacher and coach at Community House Middle School. Tyler created numerous free resources for MATHCOUNTS coaches in his role as the 2021-2022 DoD STEM Ambassador for MATHCOUNTS. Find more resources and information at **dodstem.us**.*

Basics of Bases

Warm-Up!

Try these problems before watching the lesson.

1. Rewrite 123 to show it as multiples of powers of 10.

2. Rewrite the number $12.\overline{34}$ as a common fraction.

3. What is the sum of the solutions to the equation $(2x + 3)^2 = 6x^2 + 7x + 11$? Express your answer as a common fraction.

4. How many 3-digit numbers are there?

The Problems

Take a look at the following problems and follow along as they are explained in the video.

5. a) Convert 233_5 to base 10. b) Convert $233_{5/2}$ to base 10.

6. Convert 254_6 to base 4.

7. What is the product of 52_6 and 43_6? Write your answer in base 6.

8. If $a = .\overline{4}_6$ and $b = 12.\overline{3}_5$, what is their product in base 4? Express your answer as a common fraction.

9. Assuming b is positive, what is b if $(13_b)^2 = 202_b$?

Piece It Together

Use the skills you practiced in the warm-up and strategies from the video to solve the following problems.

10. What is the sum of…
    a) 35_6 and 42_6? b) 723_8 and 462_8?

11. What is the product of 72_8 and 63_8? Write the number in base 6.

12. How many 3-digit numbers in base 6 are also 3-digit numbers in base 8?

13. What is b such that the product of 34_b and 51_b is $2{,}424_b$? Base b is a positive integer.

14. b is the smallest possible integer base for which 232_b is a square number in base 10. What is the square root of 232_b expressed in base b?

15. What is the smallest integer in base 10 that would have 5 digits in base 3/2?

Optional Extension

To extend your understanding and have a little fun with math, try the following activity.

So far, we have talked about fractional bases and positive integer bases, but there are many different types. Let's spend some time looking at negative bases. Keep in mind that each number must be represented in a unique way. This means that you can't use a negative sign in a negative base number.

- Create an addition and multiplication table for base −6. What patterns do you see or what do you notice?
- Simplify the following expressions, with all numbers in base −6:
    1. 234 + 152
    2. 342 + 425
    3. 42 × 53
    4. 123 × 45