MATHCOUNTS®

2025 State Competition
Sprint Round Problems 1–30

HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete®. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature _____ Date _____

Printed Name _____

School _____

Chapter _____

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This section of the competition consists of 30 problems. You will have 40 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the left-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

In each written round of the competition, the required unit for the answer is included in the answer blank. The plural form of the unit is always used, even if the answer appears to require the singular form of the unit. The unit provided in the answer blank is the only form of the answer that will be accepted.

Total Correct	Scorer's Initials

1. $\$$ _____ A pizza that costs \$32 is cut into 16 identical pieces, and Gil eats 3 of those pieces. What was the monetary value of the portion of pizza Gil ate?

2. _____ A standard six-sided die is constructed such that the opposite faces always sum to 7. The *multiside* is the product of the four faces that are between the top and bottom faces when it is rolled. What is the largest possible value of the *multiside*?

3. _____ A package of candies contains 3 red, 4 orange, 2 yellow, 1 green, 5 blue and 7 brown candies. What is the probability that a randomly selected candy will be green or blue? Express your answer as a common fraction.

4. _____ What is the remainder when 33^7 is divided by 5?

5. _____ What is the value of $\dfrac{5\times 44\times 333\times 2222\times 11111}{55555\times 4444\times 333\times 22\times 1}$?

6. _____ What is the value of x in the equation $\frac{1}{2} + \frac{1}{3} = \frac{1}{x}$? Express your answer as a common fraction.

7. _____ cm^2 What is the greatest possible area, in square centimeters, of a triangle with perimeter of 24 cm? Express your answer in simplest radical form.

8. _____ A pair of fair, standard six-sided dice is rolled. What is the probability that the sum of the numbers rolled is 7 or 10? Express your answer as a common fraction.

9. _____ If Nana's average on her first four exams is 78, what must she score on the fifth exam to increase her average to 81?

10. _____ km Lakeisha rode her bike from Mendelbury to Hypatiaville, passing the towns of Adaburg and then Tao Town on the way. She rode 3 km from Mendelbury to Adaburg. She rode 5 km from Tao Town to Hypatiaville. Lakeisha's return trip from Hypatiaville to Mendelbury followed the same route and was 14 km. How many kilometers did Lakeisha ride from Adaburg to Tao Town?

11. divisors

How many positive integer divisors of 60 have as factors exactly two of the numbers 2, 3 and 5?

12. _____

If $\left(\frac{x^8}{x^3}\right)^3 = x^n$, what is the value of n for $x > 1$?

13. _____

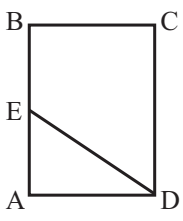
A cube that has volume 216 cm^3 is inscribed in a sphere. The volume of this sphere can be expressed in simplest radical form in terms of π as $a\pi\sqrt{b}$ cm^3. What is the value of $a + b$?

14. _____

One-tenth the sum of the positive integers from n to $n + 10$, inclusive, equals an integer s. What is the least possible value of s?

15. _____

Rectangle ABCD, shown here, has point E on segment AB such that the ratio of the area of trapezoid EBCD to triangle AED is 7:3. What is the value of the ratio AE:EB? Express your answer as a common fraction.



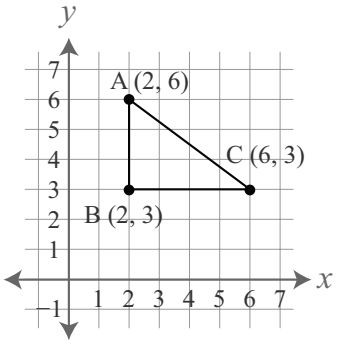
16. _____ What is the largest prime factor of $3^9 + 9^3$?

17. _____ cm The absolute difference in the areas of two concentric circles whose radii are an integer number of centimeters is 64π cm^2. What is the sum of the possible values for the radius of the larger circle?

18. _____ If x and y are distinct real numbers such that $x = y^2 - 4$ and $y = x^2 - 4$, then what is the value of $x^2 + y^2$?

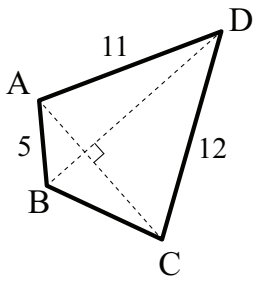
19. _____ If $5b = b^2 + 6.21$, what is the least possible value of b? Express your answer as a decimal to the nearest tenth.

20. _____ units Triangle ABC in the coordinate plane has vertices A(2, 6), B(2, 3) and C(6, 3). A line drawn through point B intersects side AC at point D. If line BD is perpendicular to side AC, what is the length of segment AD? Express your answer as a common fraction.



21. _____ inches

Convex quadrilateral ABCD has perpendicular diagonals. If AB = 5 inches, DA = 11 inches and CD = 12 inches, what is BC? Express your answer in simplest radical form.



22. _____

Let n be a positive integer less than or equal to 1000. If the last two digits of n are reversed, the resulting integer is exactly 85 percent of n. What is the sum of the possible values of n?

23. _____ ways

In how many distinguishable ways can 2 identical red marbles, 2 identical green marbles and 2 identical blue marbles be distributed among 4 labeled urns (A, B, C and D), if each marble must be placed in an urn, no urn can contain more than one marble of each color, and up to two urns may remain empty?

24. _____ integers

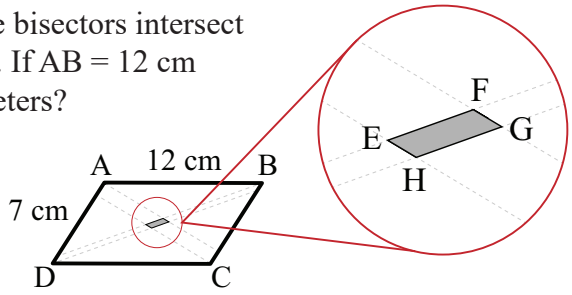
How many odd positive integers with at most four digits contain at least two even digits?

25. _____

Ten fair coins are flipped simultaneously. What is the probability that the product of the number of coins landing heads up and the number landing tails up is at least 20? Express your answer as a common fraction.

26. _____ When $\sqrt{7+4\sqrt{3}} - \sqrt{7-4\sqrt{3}}$ is written in the form $a\sqrt{3}$, what is the value of a?

27. _____ cm In parallelogram ABCD, the four angle bisectors intersect to form quadrilateral EFGH, as shown. If AB = 12 cm and BC = 7 cm, what is EG, in centimeters?



28. _____ What fraction of the permutations of the 10 letters in the word MATHCOUNTS contains a substring of 8 consecutive letters that are all different? Express your answer as a common fraction.

29. _____ Let S be the sum of all the positive integers that are factors of 999,999. What is the largest prime number that is a factors of S?

30. _____ If x and y are real numbers such that $(4 - x)(4 + y) = 2$ and $(4 + x)(4 - y) = 3$, what is the value of $(x^2 - 1)(y^2 - 1)$? Express your answer as a common fraction

Forms of Answers

The following list explains acceptable forms for answers. Coaches should ensure that Mathletes are familiar with these rules prior to participating at any level of competition. Competition answers will be scored in compliance with these rules for forms of answers.

Units of measurement are not required in answers, but they must be correct if given. When a problem asks for an answer expressed in a specific unit of measure or when a unit of measure is provided in the answer blank, equivalent answers expressed in other units are not acceptable. For example, if a problem asks for the number of ounces and 36 oz is the correct answer, 2 lbs 4 oz will not be accepted. If a problem asks for the number of cents and 25 cents is the correct answer, \$0.25 will not be accepted.

The plural form of the units will always be provided in the answer blank, even if the answer appears to require the singular form of the units.

Geometric figures may not be drawn to scale and lengths of geometric figures should be assumed to be measured in "units" unless otherwise stated.

All answers must be expressed in simplest form. A "common fraction" is to be considered a fraction in the form $\pm\frac{a}{b}$, where a and b are natural numbers and GCF(a, b) = 1. In some cases the term "common fraction" is to be considered a fraction in the form $\frac{A}{B}$, where A and B are algebraic expressions and A and B do not share a common factor. A simplified "mixed number" ("mixed numeral," "mixed fraction") is to be considered a fraction in the form $\pm N\frac{a}{b}$, where N, a and b are natural numbers, $a < b$ and GCF(a, b) = 1. Examples:

Problem: What is 8 ÷ 12 expressed as a common fraction? *Answer:* $\frac{2}{3}$ *Unacceptable:* $\frac{4}{6}$

Problem: What is 12 ÷ 8 expressed as a common fraction? *Answer:* $\frac{3}{2}$ *Unacceptable:* $\frac{12}{8}$, $1\frac{1}{2}$

Problem: What is the sum of the lengths of the radius and the circumference of a circle with diameter $\frac{1}{4}$ unit expressed as a common fraction in terms of π? *Answer:* $\frac{1+2\pi}{8}$

Problem: What is 20 ÷ 12 expressed as a mixed number? *Answer:* $1\frac{2}{3}$ *Unacceptable:* $1\frac{8}{12}$, $\frac{5}{3}$

Ratios should be expressed as simplified common fractions unless otherwise specified. Examples:

Acceptable Simplified Forms: $\frac{7}{2}$, $\frac{3}{\pi}$, $\frac{4-\pi}{6}$ *Unacceptable:* $3\frac{1}{2}$, $\frac{\frac{1}{4}}{3}$, 3.5, 2:1

Radicals must be simplified. A simplified radical must satisfy: 1) no radicands have a factor which possesses the root indicated by the index; 2) no radicands contain fractions; and 3) no radicals appear in the denominator of a fraction. Numbers with fractional exponents are *not* in radical form. Examples:

Problem: What is $\sqrt{15} \times \sqrt{5}$ expressed in simplest radical form? *Answer:* $5\sqrt{3}$ *Unacceptable:* $\sqrt{75}$

Answers to problems asking for a response in the form of a dollar amount or an unspecified monetary unit (e.g., "How many dollars...," "How much will it cost...," "What is the amount of interest...") should be expressed in the form (\$) *a.bc* or *a.bc* (dollars), where *a* is an integer and *b* and *c* are digits. The *only* exceptions to this rule are when a is zero, in which case it may be omitted, or when b and c both are zero, in which case they both may be omitted. Answers in the form (\$) *a.bc* or *a.bc* (dollars) should be rounded to the nearest cent unless otherwise specified. Examples:

Acceptable Forms: 2.35, 0.38, .38, 5.00, 5 *Unacceptable:* 4.9, 8.0

Do not make approximations for numbers (e.g., π, $\frac{2}{3}$, $5\sqrt{3}$) in the data given or in solutions unless the problem says to do so.

Do not perform any intermediate rounding (other than the "rounding" a calculator does) when calculating solutions. All rounding should be done at the end of the computation process.

Scientific notation should be expressed in the form $a \times 10^n$ where a is a decimal, $1 \le |a| < 10$, and n is an integer. Examples

Problem: What is 6895 expressed in scientific notation? *Answer:* 6.895×10^3

Problem: What is 40,000 expressed in scientific notation? *Answer:* 4×10^4 *or* 4.0×10^4

An answer expressed to a greater or lesser degree of accuracy than called for in the problem will not be accepted. Whole-number answers should be expressed in their whole-number form. Thus, 25.0 will not be accepted for 25, and 25 will not be accepted for 25.0.